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Dropping an impurity into a Chern insulator: A polaron view on topological matter

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Introduction

Mobile impurities in a quantum bath play a fundamental role in a wide range of systems (metals, semiconductors, Helium mixtures, high-T_c superconductors, …)

Here we study an impurity particle interacting with a Fermi gas in a Chern-insulating state. The interaction leads to the formation of an exotic polaron, consisting of a coherent superposition of the "trivial" impurity and the surrounding topological cloud.

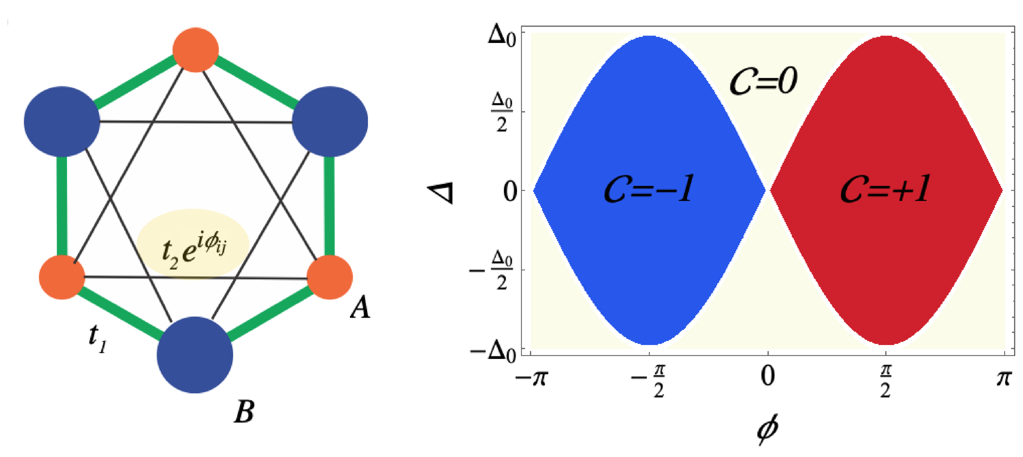
We characterize this composite object by calculating the *polaronic transverse conductivity*, i.e., the transverse drag exerted by the dressing cloud on the impurity, using diagrammatic as well as variational methods.

The polaron is shown to inherit some of the topological properties of the Chern insulator through genuine interaction effects.

System

- A **single impurity** (↓) on a *graphene* lattice.
- Fermi gas (↑) filling the lowest band of a *Haldane* lattice (graphene lattice + time-reversal breaking NNN hopping + A/B energy offset) so that it forms a **topological band insulator**.

$$\hat{H}_0 = -t_1 \sum_{\sigma=\uparrow,\downarrow}\sum_{\langle i,j\rangle} \hat{c}^\dagger_{iA\sigma}\hat{c}_{jB\sigma} - t_2 \sum_{\langle\langle i,j\rangle\rangle} e^{i\phi_{ij}}\hat{c}^\dagger_{iA\uparrow}\hat{c}_{jB\uparrow} + \text{h.c.} + \Delta\sum_i(\hat{c}^\dagger_{iA\uparrow}\hat{c}_{iA\uparrow} - \hat{c}^\dagger_{iB\uparrow}\hat{c}_{iB\uparrow}) = \sum_{\mathbf{k}\alpha\sigma}\varepsilon_{\sigma\alpha}(\mathbf{k})\hat{\gamma}^\dagger_{\mathbf{k}\alpha\sigma}\hat{\gamma}_{\mathbf{k}\alpha\sigma}.$$



- Contact interaction between impurity and bath:

$$\hat{H}_{\text{int}} = g\sum_i\sum_{s=A,B}\hat{c}^\dagger_{is\uparrow}\hat{c}^\dagger_{is\downarrow}\hat{c}_{is\downarrow}\hat{c}_{is\uparrow} = \frac{g}{N}\sum_{\substack{\mathbf{k}\mathbf{k}'\mathbf{q}\\ \alpha\beta\alpha'\beta'}} W^{\alpha\beta}_{\alpha'\beta'}(\mathbf{k},\mathbf{k}',\mathbf{q})\hat{\gamma}^\dagger_{\mathbf{k}+\mathbf{q}\alpha\uparrow}\hat{\gamma}^\dagger_{\mathbf{k}'-\mathbf{q}\alpha'\downarrow}\hat{\gamma}_{\mathbf{k}'\beta'\downarrow}\hat{\gamma}_{\mathbf{k}\beta\uparrow}$$

Variational treatment

- Interactions lead to the creation of a *Fermi polaron*
- Chevy Ansatz: $|\psi_0\rangle = \sqrt{Z_0}|\varphi_0\rangle + \sum_{\mathbf{Q},\mathbf{q},\alpha} M_{\mathbf{Q},\mathbf{q},\alpha}|\varphi_{\mathbf{Q},\mathbf{q},\alpha}\rangle$

  non-interacting ground state: $|\varphi_0\rangle$

  non-interacting excited. state: $|\varphi_{\mathbf{Q},\mathbf{q},\alpha}\rangle \equiv \hat{\gamma}^\dagger_{\mathbf{q}2\uparrow}\hat{\gamma}^\dagger_{\mathbf{Q}-\mathbf{q}\alpha\downarrow}\hat{\gamma}_{\mathbf{Q}1\uparrow}\hat{\gamma}_{\mathbf{0}1\downarrow}|\varphi_0\rangle$
- Minimize $\langle\hat{H}_0 + \hat{H}_{\text{int}} - E\rangle$ to find Z_0 and $\{M_{\mathbf{Q},\mathbf{q},\alpha}\}$
- However, the topological properties of the underlying substrate will not affect drastically the "usual" quasiparticle properties, such as the polaron energy or residue.

External force & transverse current

Non-trivial effects due to the topology of the underlying substrate appear in the *transverse response of the polaron to an external force*.

We consider a uniform force, acting on both the impurity and the bath: $\mathbf{F}(\mathbf{r}) = -\nabla V(\mathbf{r})$ (with $\mathbf{F}(\mathbf{r}) = -F_y\mathbf{e}_y$) generating a perturbation $\hat{H}'(t) = \int d^2r V(\mathbf{r})\hat{\rho}(\mathbf{r},t)$

Interactions can generate a non-zero transverse (Hall) current on the impurity: $\langle\hat{j}_{x\downarrow}\rangle = \sigma^P_{xy}\cdot(-F_y)$

Within linear response, the Hall polaron conductivity can be written in terms of the current-current correlation function:

$$\sigma^P_{xy} = \lim_{\omega\to 0} -\frac{\mathcal{P}_{xy}(\omega)}{i\omega}$$

where $\mathcal{P}_{xy}(t-t') = -iN\theta(t-t')\langle\psi_0|[\hat{j}_{x\downarrow}(t), \hat{j}_{y\uparrow}(t') + \hat{j}_{y\downarrow}(t')]|\psi_0\rangle$

Composite Berry curvature

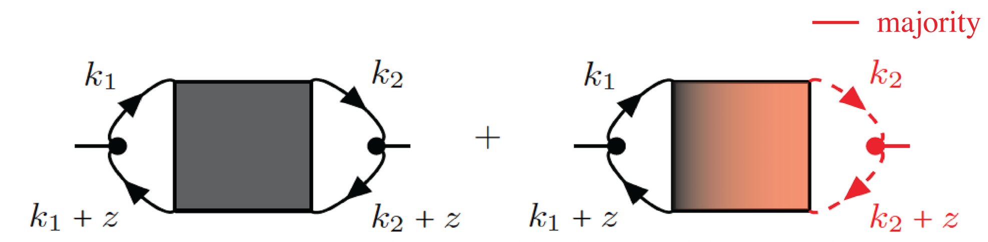
- The *Berry curvature* is an essential tool to understand non-interacting Chern insulators.
- Likewise, we can here introduce a *composite Berry curvature* for the polaron, writing $\sigma^P_{xy} = \sum_{\mathbf{Q}}\mathcal{B}_{\downarrow\uparrow}(\mathbf{Q})$

$$\mathcal{B}_{\sigma\sigma'}(\mathbf{Q}) = -i\sum_{\mathbf{k}_1,\mathbf{k}_2,\mathbf{q}}\sum_{\alpha,\beta,\alpha',\beta',\alpha''}\frac{\langle\psi_0|\hat{\mathcal{J}}^x_{\sigma\alpha\beta}(\mathbf{k}_1)|\psi_{\mathbf{Q},\mathbf{q},\alpha''}\rangle\langle\psi_{\mathbf{Q},\mathbf{q},\alpha''}|\hat{\mathcal{J}}^y_{\sigma'\alpha'\beta'}(\mathbf{k}_2)|\psi_0\rangle - x\leftrightarrow y}{(E_0 - E_{\mathbf{Q},\mathbf{q}})^2}$$

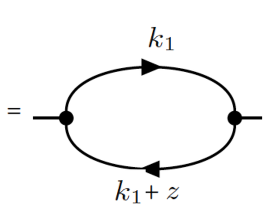
- $\mathcal{B}_{\sigma\sigma'}(\mathbf{Q})$ describes the Berry curvature of an excitation involving particles σ and σ' with total momentum ***Q***.
- $\mathcal{B}_{\uparrow\uparrow}(\mathbf{Q})$ is the usual curvature of the Haldane model.

Diagrammatic calculation

Transverse conductivity of the polaron:

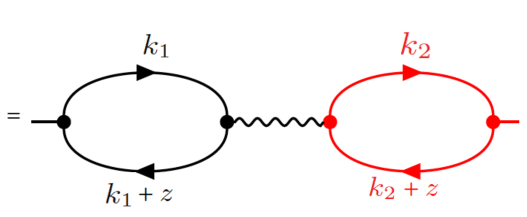


0th order: $\mathcal{P}^{(0)}_{xy}(z) =$



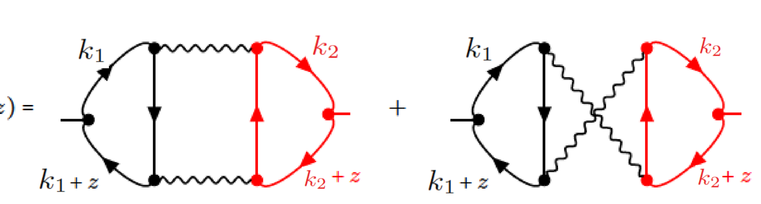
vanishes for a single impurity

1st order: $\mathcal{P}^{(1)}_{xy}(z) =$



decoupled sums —> zero contribution

2nd order: $\mathcal{P}^{(2)}_{xy}(z) =$

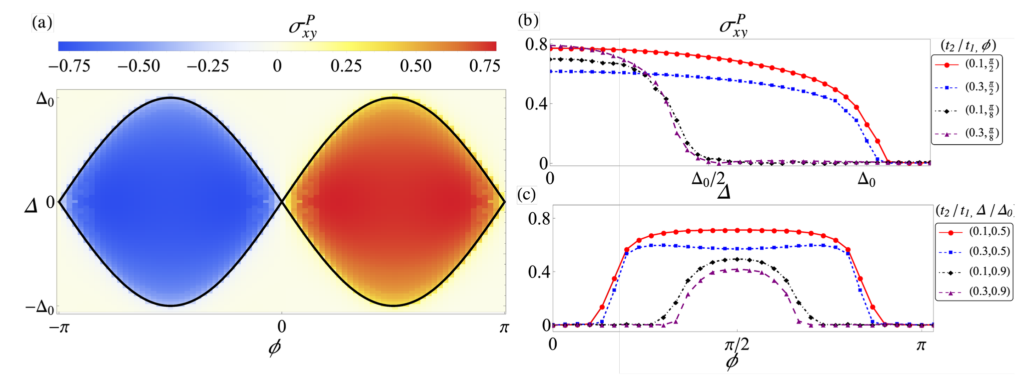


$$\mathcal{P}_{xy}(\omega) = g^2\sum_{k_1k_2k_3}\mathcal{J}^x_{\downarrow\beta'\alpha'}(\mathbf{k}_2)\mathcal{G}_{\downarrow\beta'}(k_2+\omega)\mathcal{G}_{\downarrow\alpha'}(k_2)\mathcal{G}_{\downarrow\kappa'}(k_3+k_2)\Big[W^{\beta\kappa}_{\kappa'\beta'}(\mathbf{k}_1+\mathbf{k}_3,\mathbf{k}_2,-\mathbf{k}_3)W^{\kappa\alpha}_{\alpha'\kappa'}(\mathbf{k}_1,\mathbf{k}_3+\mathbf{k}_2,\mathbf{k}_3)\times \mathcal{G}_{\uparrow\kappa}(k_1+k_3) + W^{\beta\kappa}_{\alpha'\kappa'}(\mathbf{k}_1-\mathbf{k}_3,\mathbf{k}_3+\mathbf{k}_2,\mathbf{k}_3)W^{\kappa\alpha}_{\kappa'\beta'}(\mathbf{k}_1,\mathbf{k}_2,-\mathbf{k}_3)\mathcal{G}_{\uparrow\kappa}(k_1-k_3+\omega)\Big]\mathcal{G}_{\uparrow\alpha}(k_1)\mathcal{G}_{\uparrow\beta}(k_1+\omega)\mathcal{J}^y_{\uparrow\beta\alpha}(\mathbf{k}_1)$$

An identical result can equivalently be obtained evaluating the composite Berry curvature in terms of the Chevy Ansatz, expanded to 2nd order in g.

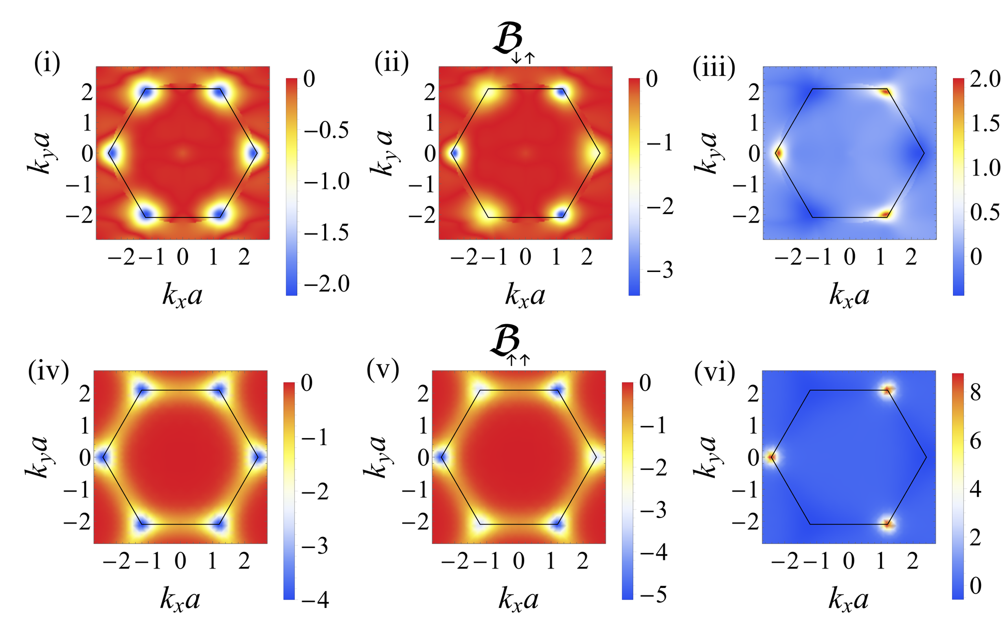
Results

- The transverse conductivity of the polaron scales as g^2:



σ^P_{xy} in units of g^2/Ng_0^2 with $g_0 = (2\pi a)^2/(3\sqrt{3}/2)t_1$, computed for $t_2 = 0.1t_1$

- σ^P_{xy} is qualitatively very similar to the Chern number $\mathcal{C}$:
  - it vanishes whenever $\mathcal{C} = 0$
  - however, it is not quantized.
- Composite Berry curvature of the polaron:



(i)–(iii) The composite Berry curvature $\mathcal{B}_{\downarrow\uparrow}(\mathbf{Q})$ of the polaron for (i) $(\phi,\Delta) = (-\pi/2, 0)$, (ii) $(\phi,\Delta) = (-\pi/2, \Delta_0/3)$, and (iii) $(\phi,\Delta) = (-\pi/2, 5\Delta_0/3)$. (iv)–(vi) below show the corresponding Berry curvature $\mathcal{B}_{\uparrow\uparrow}(\mathbf{Q})$ for the majority for the same values of (ϕ,Δ).

- $\mathcal{B}_{\downarrow\uparrow}$ closely mimics $\mathcal{B}_{\uparrow\uparrow}$: the *geometric properties* of the substrate are faithfully mapped on the polaron!

Experimental remarks and conclusions

- The Haldane model has been realized, and its Berry curvature measured, using ultracold atoms [1-3].
- Transverse displacement of the impurity: $\delta x_\downarrow = v_{x\downarrow}\tau = j_{x\downarrow}\tau/n_\downarrow = \sigma^P_{xy}F_y\tau/n_\downarrow$
- With $\sigma^P_{xy} \simeq 0.8g^2/Ng_0^2$, $\tau \approx 50\pi/t_1$, $F_y \approx 0.3t_1/a$, and $g \ll 6t_1$, one finds a significant drift $\delta x_\downarrow \approx 5a$.
- If the force acted on the impurity only, $\sigma^P_{xy} \propto g^4$.
- Related work investigated impurities strongly-bound to quasiparticles in FQH and 1D chiral models [4, 5]

References

[1] G. Jotzu *et al.*, *Experimental realization of the topological Haldane model with ultracold fermions*, Nature **515**, 237 (2014).

[2] L. Duca *et al.*, *An Aharonov-Bohm interferometer for determining Bloch band topology*, Science **347**, 288 (2015).

[3] N. Fläschner *et al.*, *Experimental reconstruction of the Berry curvature in a Floquet Bloch band*, Science **352**, 1091 (2016).

[4] F. Grusdt *et al.*, *Interferometric measurements of many-body topological invariants using mobile impurities*, Nat. Comm. **7**, 11994 (2016).

[5] F. Grusdt, N. Yao, and E. Demler, *Topological polarons, quasiparticle invariants, and their detection in one-dimensional symmetry-protected phases*, Phys. Rev. B **100**, 075126 (2019).

[this work] A. Camacho-Guardian, P. Massignan, N. Goldman, and G. Bruun, *Dropping an impurity into a Chern insulator: A polaron view on topological matter*, Phys. Rev. B **99**, 081105(R) (2019).

Acknowledgments

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